Strings in a space with tensor central charge coordinates

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New string models in $D = 4$ space-time extended by tensor central charge coordinates z_{mn} are constructed. We use the z_{mn} coordinates to generate string tension using a minimally extended string action linear in dz^{mn}. It is shown that the presence of z_{mn} lifts the light-like character of the tensionless string worldsheet and the degeneracy of its induced metric. We analyse the equations of motion and find a solution of the string equations in the generalized D=(4+6)-dimensional space $X^{\mathcal{M}} = (x^m, z^{mn})$ with z_{mn} describing a spin wave process. A supersymmetric version of the proposed model is formulated.

1. Introduction

Development of noncommutative geometry ideas [1], [2], [3], [4] have resulted in the discovery of Noncommutative Open String theory [5], [6], [7]. An interesting feature of the NCOS theory is the appearence of a critical value of the electric field [8] for which the effective string tension becomes equal to zero. So, the question arises as to the connection of this theory to the standard theory of tensionless strings [9],[10] and branes [11]. This question is further motivated by the results of papers [12], [13], where a Born-Infeld Dp-brane action was constructed and studied in the limit of zero tension. It was shown in [13] that the generalized gauge invariant Born-Infeld 2-form $\mathcal{F} = B + (\frac{\gamma}{2\pi\alpha'})^{-1}F$ splits into two mutually orthogonal parts and one of them lies in the tangent plane spanned by the vielbein components e_i^0 and e_i^1 of tensionless Dp-brane. For the case $B = 0$ the electric field E of the Dp-brane becomes constant [5] $E = \frac{\gamma}{\pi\alpha'}$ and directed along e_i^1. This observation is related to the character of the dynamics of tensionless Dp-brane [12] which is reduced to the dynamics of an effective string stretched along the e_i^1 direction. These two facts lead to the conclusion that the tension of an effective open string (and Dp-brane) embedded into the Dp-brane is entirely compensated by the critical electric field $E = \frac{\gamma}{\pi\alpha'}$ pulling apart the charges attached to the string's ends. Therefore, the question about the noncommutative nature of the space-time coordinates in string/M- theory has to be intrincically related to the question on the nature of the tension of extended objects.

Some mechanisms for the generation of tension have been discussed in [15], [18], [19], [16], [17] and they imply that tension is created by the interactions of a tensionless string/brane with some additional fields or coordinates. Selfconsistency and completeness of this picture suggest that the additional field or coordinate should be an object intrinsic to string/M-theory.

From this point of view the coordinates corresponding to the tensorial central charges $Z_{m_1,m_2,\ldots,m_p}$ are interesting. The point is that the central charges are connected with p-branes as solitonic solutions of supergravity equations and modify the Poincare superalgebra to the form [20], [21], [22], [23], [24], [25]

$$\{Q_\alpha, Q_\beta\} = (\gamma^m C^{-1})_{\alpha\beta} P_m + \sum_p \left(\gamma^{m_1,m_2,\ldots,m_p} C^{-1}\right)_{\alpha\beta} Z_{m_1,m_2,\ldots,m_p}$$

A non-trivial role of the new coordinates corresponding to the p-form generators $Z_{m_1,m_2,\ldots,m_p}$

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[5]Here γ is a parameter characterizing the background metric [14].

was emphasized in [26], [27], [28], [29] and advanced further in [30], [31], [32], where new superalgebras were constructed. The string/M-theory approach [33] considers the extended superalgebras [34] of Dp-branes [35] to be connected with the non-perturbative string dynamics.

Moreover, the central charge carried by the BPS brane/string preserving 1/2 of the N=1 supersymmety [21] appears in QCD and is associated with the domain wall created by the gluino condensate [36] (see also [37] and refs. there) and spontaneous breakdown of the discrete chiral symmetry [38]. Recently it was shown that the intersection of domain walls leads to creation of configurations preserving 1/4 [39] and 3/4 [40] of the centrally extended supersymmetries. The possibility for preserving 3/4 supersymmetries was earlier noted [41] to be one of the solutions for the superparticle [42] moving in a superspace extended by the coordinates of the tensorial central charge. In [43] the combinations of momentum and domain-wall charges corresponding to BPS states preserving $1/2, 1/4$ and $3/4$ of $D = 4$ $N = 1$ supersymmetry were constructed and it was proved was that the Wess-Zumino model does not admit any classical configurations with 3/4 supersymmetry.

A unified geometric approach to a description of superbranes was developed in [44] where new reasons for an enlargement of superspace were given.

Here we propose new models for strings moving in $D = 4$ space-time extended by six real coordinates z_{mn} corresponding to the tensor central charges Z_{mn}. The action in our model is a natural generalization of a new twistor like representation [17] for the Nambu-Goto and tensionless string actions. The suggested model admits $N = 1$ $D = 4$ supersymmetrization and therefore may be effectively treated as a bosonic sector of superstrings moving in the generalized $D = (4 + 6)-$dimensional space-time.

To make the dynamical role of the z_{mn} coordinates clear the simplest case of the tensionless string in a twistor-like formulation [17] is studied here. We find that even a minimal inclusion of the z_{mn} coordinates, linear in their derivatives, lifts the light-like character of the tensionless string worldsheet and removes the degeneracy of the worldsheet metric. This is a hint of string tension being generated. We solve the string equations together with the system of integrability conditions and find a solution for x^m and z^{mn} which can be interpreated as a solution of the wave equation $\ddot{\mathrm{X}}_{\mathcal{M}} - X''_{\mathcal{M}} = 0$ describing a string in the extended D=10 space with the coordinates $X^{\mathcal{M}} = (x^m, z^{mn})$. In this solution the x^m coordinates have no transverse oscillations. The solution for the z^{mn} coordinates implies the appearance of spin structure distributed along the string and a spin wave [45],[46] process related to this structure. A supersymmetric version of the proposed string model is formulated.

2. String action in $D = 4$ space-time with tensor central charge coordinates

To describe the string dynamics we start from a twistor-like representation of the tensile/tensionless string action [17]

$$S = \kappa \int (p_{mn}\, dx^m \wedge dx^n + \Lambda), \tag{1}$$

which includes a local bivector $p_{mn}(\tau, \sigma)$ composed of the local Newman-Penrose dyads attached to the worldsheet and where the $\Lambda-$term fixes the orthonormality constraint for the spinorial dyads (or twistor like variables).

For the case of tensionless strings the Lagrange multiplier Λ=0 and $p_{mn}(\tau, \sigma)$ is a null bivector defined by the condition

$$p_{mn}p^{mn} = 0 \quad , \qquad \eta_{mn} = (- + ++), \tag{2}$$

which implies

$$\begin{aligned} p_{mn}(\tau, \sigma) &= i\bar{U}\gamma_{mn}U \\ &= 2i[u^\alpha(\sigma_{mn})_\alpha^\beta u_\beta + \bar{u}_{\dot\alpha}(\tilde\sigma_{mn})^{\dot\alpha}_{\dot\beta}\bar{u}^{\dot\beta}], \end{aligned} \tag{3}$$

where U_a is a Majorana bispinor

$$U_a = \begin{pmatrix} u_\alpha \\ \bar{u}^{\dot\alpha} \end{pmatrix}, \ \gamma_{mn} = \frac{1}{2}[\gamma_m, \gamma_n],$$
$$\sigma_{mn} = \frac{1}{4}(\sigma_m\tilde{\sigma_n} - \sigma_n\tilde{\sigma_m}). \tag{4}$$

For the tensile string the bivector $p_{mn}(\tau,\sigma)$ may be represented as a sum of two null bivectors $p^{(+)}_{mn}$ and $p^{(-)}_{mn}$ [17]

$$p_{mn} = p^{(+)}_{mn} + p^{(-)}_{mn} = i\,[\bar{U}\gamma_{mn}U + \bar{V}\gamma_{mn}V], \qquad (5)$$

where $V_a = \begin{pmatrix} v_\alpha \\ \bar{v}^{\dot\alpha} \end{pmatrix}$ is the second component of the Newman-Penrose dyads $(u_\alpha(\tau,\sigma), v_\alpha(\tau,\sigma))$

$$u^\alpha v_\alpha = 1, \quad u^\alpha u_\alpha = v^\alpha v_\alpha = 0 \qquad (6)$$

and the $\Lambda(\tau,\sigma)-$term is

$$\Lambda(\tau,\sigma) = \lambda(u^\alpha v_\alpha - 1) - \bar\lambda(\bar{u}^{\dot\alpha}\bar{v}_{\dot\alpha} - 1). \qquad (7)$$

The action (1) may be rewritten in an equivalent spinor form

$$S = i\kappa \int [\, p^{ab}\, dx_{ae} \wedge dx_{db} C^{ed} + \Lambda\,], \qquad (8)$$

where $C^{ed} = (\gamma^0)^{ed}$ is the charge conjugation matrix in the Majorana representation and p^{ab} is a symmetric local spin-tensor. In the general case p^{ab} may be represented as a bilinear combination of the Majorana bispinors U_a and V_a.

$$p^{ab} = \alpha\, U^a U^b + \beta\, V^a V^b + \varrho\,(U^a V^b + U^b V^a) \quad (9)$$

with arbitrary coefficients α, β and ϱ.

The representation (8) includes an interesting object - the differential 2-form of the worldsheet area element ξ_{ab} in the spinor representation

$$\xi_{ab} = \xi_{ba} = C^{ed}\, dx_{ae} \wedge dx_{db}, \qquad (10)$$

where

$$dx_{ab} = (\gamma_m\, C^{-1})_{ab}\, dx^m \qquad (11)$$

and ξ_{ab} is a symmetric spin-tensor 2-form.

To include the real antisymmetric central charge coordinates z_{mn} we consider the following extension of x_{ab}

$$x_{ab} \longrightarrow Y_{ab} = x^m\,(\gamma_m\, C^{-1})_{ab} + i z_{mn}\,(\gamma^{mn}\, C^{-1})_{ab}, \qquad (12)$$

used earlier in [42] for the case of superparticles. As a result of (12) the area element ξ_{ab} and the action (8) are replaced by

$$\xi_{ab} \longrightarrow \Xi_{ab} = dY_{al} \wedge dY^{l}{}_{b}. \qquad (13)$$

and

$$S = i\kappa \int (p^{ab} d\,\Xi_{ab} + \Lambda\,) = i\kappa \int (p^{ab}\, dY_{ae} \wedge dY_{db}\, C^{ed} + \Lambda\,). \qquad (14)$$

Our goal is to study the generalized action (14) and to this end we note that the generalized area element Ξ_{ab} (13) is

$$\Xi_a{}^b = (\, dx_m \wedge dx_n - 8 dz_{ml} \wedge dz_n{}^l)(\gamma^{mn})_a{}^b - 4i\, dx^l \wedge dz_{lm} (\gamma^m)_a{}^b. \quad (15)$$

Using properties of the $\gamma-$matrix algebra, the action S (14) takes the form

$$S = i\kappa \int \{\, [\, (\, dx_m \wedge dx_n - 8 dz_{ml} \wedge dz_n{}^l)\gamma^{mn} - 4i\, dx^l \wedge dz_{lm} \gamma^m]_a{}^b\, p^a{}_b + \Lambda\, \}. \quad (16)$$

In (16) x_m and z_{mn} appear on equal footing. As a first investigation of the model, however, we shall drop the quadratic $z-$term and study the minimally extended action

$$S = i\kappa \int [(\, dx_m \wedge dx_n \gamma^{mn} - 4i\, dx^l \wedge dz_{lm} \gamma^m)_a{}^b\, p^a{}_b + \Lambda], \qquad (17)$$

We may think of this action as the action (16) in a certain limit or as a model in its own right. In particular we shall study the minimal extension of the tensionless string.

3. Tensionless string in the space with central charge coordinates $z_{\alpha\beta}$

The action for the tensionless string (1), (3) minimally extended by central charge coordinates is (17) with $\Lambda = 0$ and p^{ab} on the form [17]

$$p^{ab} = U^a U^b. \qquad (18)$$

It reads

$$S = i \int U^a(\, dx_m \wedge dx_n \gamma^{mn} - 4i dx^l \wedge dz_{lm} \gamma^m)_a{}^b U_b, \qquad (19)$$

where κ is included in a redefinition of x_m and z_{lm} to make all variables (19) dimensionless.

The dynamics generated by the action S (19) will in general break the light-like character of the string worldsheet [9], [10]. To study the dynamics S (19) we use the Weyl representation for z_{mn} , where S (19) takes the form

$$S = 2i\int\{\, dx_m\wedge dx_n(\, u\sigma^{mn}u + \bar{u}\tilde{\sigma}^{mn}\bar{u}) \\ -dx^l\wedge[u_\alpha dz^{\alpha\beta}(\sigma_l\bar{u})_\beta - \bar{u}_{\dot\alpha}d\bar{z}^{\dot\alpha\dot\beta}(u\sigma^l)_{\dot\beta}\,]\}. \quad (20)$$

To analyse the $u-$equations of motion we define the 2-forms

$$\Sigma_\alpha{}^\beta \equiv dx_m\wedge dx_n(\sigma^{mn})_\alpha{}^\beta = \frac{1}{2}\,dx_{\alpha\dot\lambda}\wedge d\tilde{x}^{\dot\lambda\beta} \\ \tilde{\Sigma}^{\dot\beta}{}_{\dot\alpha} \equiv -(\Sigma_\alpha{}^\beta)^* = dx_m\wedge dx_n(\tilde{\sigma}^{mn})^{\dot\beta}{}_{\dot\alpha} \\ = \frac{1}{2}\,d\tilde{x}^{\dot\beta\lambda}\wedge dx_{\lambda\dot\alpha}, \quad (21)$$

and the antihermitian 2-form $\Omega_{\alpha\dot\beta}$

$$\Omega_{\alpha\dot\beta} \equiv -8i(dz_{ml}\wedge dx^l)(\sigma^m)_{\alpha\dot\beta} \\ = 2[(dz_\alpha{}^\lambda\wedge dx_{\lambda\dot\beta} + dx_{\alpha\dot\lambda}d\bar{z}^{\dot\lambda}{}_{\dot\beta}], \\ (\Omega_{\alpha\dot\beta})^* = -\Omega_{\beta\dot\alpha}. \quad (22)$$

Then the action (20) becomes

$$S = 2i\int(u^\alpha\Sigma_\alpha{}^\beta u_\beta + \bar{u}_{\dot\beta}\tilde{\Sigma}^{\dot\beta}{}_{\dot\alpha}\bar{u}^{\dot\alpha} + \frac{1}{2}u^\alpha\Omega_{\alpha\dot\beta}\bar{u}^{\dot\beta}) \quad (23)$$

and the equations of motion for the dyad u_α are

$$\Sigma_\alpha{}^\beta u_\beta + \frac{1}{4}\Omega_{\alpha\dot\beta}\bar{u}^{\dot\beta} = 0, \\ \bar{u}_{\dot\beta}\tilde{\Sigma}^{\dot\beta}{}_{\dot\alpha} + \frac{1}{4}u^\beta\Omega_{\beta\dot\alpha} = 0. \quad (24)$$

Under the assumption that $\Sigma_\alpha{}^\beta$ is non-degenerate (24) is equivalent to

$$(\Sigma_\alpha{}^\beta - \frac{1}{16}V_\alpha{}^\beta)u_\beta = 0, \quad (25)$$

where the 2-form $V_\alpha{}^\lambda\varepsilon_{\lambda\beta}$ is the symmetric traceless 2-form

$$V_\alpha{}^\beta \equiv (\Omega\tilde{\Sigma}^{-1}\varepsilon\Omega^*\varepsilon)_\alpha{}^\beta, \\ V_\alpha{}^\lambda\varepsilon_{\lambda\beta} = V_\beta{}^\lambda\varepsilon_{\lambda\alpha}. \quad (26)$$

Eqs. (24) are equivalent to the equation

$$\Sigma_\alpha{}^\beta - \frac{1}{16}V_\alpha^\beta = -Qu_\alpha u^\beta, \quad (27)$$

where Q is an arbitrary 2-form. All the matrices in Eq. (27) are symmetric and their determinants are given by the relation

$$\det(\mathcal{A}_\alpha{}^\beta) = -\frac{1}{2}Tr(\mathcal{A}^2). \quad (28)$$

Using (28) Eqs. (27) yield

$$\det\Sigma = (\frac{1}{16})^2\det V + \frac{1}{16}Qu_\alpha V_\beta^\alpha u^\beta. \quad (29)$$

The definition (21) of $\Sigma_\alpha{}^\beta$ gives

$$\det\Sigma = \frac{1}{2}(\,dx^m\wedge dx^n)(\,dx_m\wedge dx_n) \quad (30)$$

and we conclude that the induced worldsheet metric is not in general degenerate, i.e.

$$\det\Sigma \neq 0. \quad (31)$$

4. Solution of the equations of motion

To analyse the total set of string equations we start from the Weyl representation for S (20)

$$S = i\int[(u^\alpha dx_{\alpha\dot\lambda}\wedge d\tilde{x}^{\dot\lambda\beta}u_\beta + \bar{u}_{\dot\alpha}d\tilde{x}^{\dot\alpha\lambda}\wedge dx_{\lambda\dot\beta}\bar{u}^{\dot\beta}) \\ +2(u_\alpha dz^{\alpha\beta}\wedge dx_{\beta\dot\lambda}\bar{u}^{\dot\lambda} - \bar{u}_{\dot\alpha}d\tilde{z}^{\dot\alpha\dot\beta}\wedge u^\lambda dx_{\lambda\dot\beta})], \quad (32)$$

The variation of S in (32) with respect to $z^{\dot\alpha\dot\beta}$ and $x_{\alpha\dot\beta}$ gives

$$d\wedge(u_\alpha(dx\bar{u})_\beta + u_\beta(dx\bar{u})_\alpha) = 0 \quad (33)$$

and

$$d\wedge[-u^\alpha(d\tilde{x}u)^{\dot\beta} + \bar{u}^{\dot\beta}(\bar{u}d\tilde{x})^\alpha \\ +\bar{u}^{\dot\beta}(udz)^\alpha - u^\alpha(ud\bar{z})^{\dot\beta}] = 0, \quad (34)$$

respectively. (For simplicity we consider closed strings.) To solve Eqs. (33), (34) and (24) we will use Cartan's method (applied in [15,17] for the solution of string dynamics). To this end recall that the spinors u_α, v_α and $\bar{u}_{\dot\alpha}$, $\bar{v}_{\dot\alpha}$ form a local spinor frame moving along string's worldsheet which may be used to build a local vector frame with

$$u_\alpha\bar{u}_{\dot\alpha},\ u_\alpha\bar{v}_{\dot\alpha} + v_\alpha\bar{u}_{\dot\alpha},\ v_\alpha\bar{v}_{\dot\alpha},\ i(u_\alpha\bar{v}_{\dot\alpha} - v_\alpha\bar{u}_{\dot\alpha}) \quad (35)$$

as basis elements. The independent differentials $dx_{\alpha\dot\alpha}$ and $dz_{\alpha\beta}$ may be expanded in these basis elements as follows:

$$
\begin{aligned}
dx_{\alpha\dot\alpha} &= x^{(u)}u_\alpha\bar u_{\dot\alpha} + x^{(v)}v_\alpha\bar v_{\dot\alpha}\\
&\quad +x^{(+)}(u_\alpha\bar v_{\dot\alpha} + v_\alpha\bar u_{\dot\alpha})\\
&\quad +ix^{(-)}(u_\alpha\bar v_{\dot\alpha} - v_\alpha\bar u_{\dot\alpha}),\\
dz_{\alpha\beta} &= \varsigma^{(u)}u_\alpha u_\beta + \varsigma^{(v)}v_\alpha v_\beta\\
&\quad +\varsigma d u_\alpha v_\beta + u_\beta v_\alpha),\\
d\bar z_{\dot\alpha\dot\beta} &= \bar\varsigma^{(u)}\bar u_{\dot\alpha}\bar u_{\dot\beta} + \bar\varsigma^{(v)}\bar v_{\dot\alpha}\bar v_{\dot\beta}\\
&\quad +\bar\varsigma(u_{\dot\alpha}\bar v_{\dot\beta} + u_{\dot\beta}\bar v_{\dot\alpha})
\end{aligned}
\tag{36}
$$

(this defines $x^{(u)}, x^{v)}, x^{(+)}, x^{(-)}, \varsigma^{(u)}, \varsigma^{(v)}$ and ς). Substituting the expansions (36) into Eqs. (24) we get a system of equations

$$
\begin{aligned}
x^{(v)} \wedge x^{(+)} &= 0,\\
x^{(v)} \wedge (x^{(-)} - \varsigma_I)&\\
-\varsigma_I^{(v)} \wedge (x^{(+)} + x^{(-)}) &= 0,\\
x^{(u)} \wedge x^{(v)} - 2\varsigma_R \wedge x^{(+)}&\\
+\varsigma_R^{(u)} \wedge x^{(v)} + \varsigma_R^{(v)} \wedge x^{(u)} &= 0,\\
2x^{(-)} \wedge x^{(+)} - \varsigma_I^{(u)} \wedge x^{(v)}&\\
+\varsigma_I^{(v)} \wedge x^{(u)} + 2\varsigma_R \wedge x^{(-)} &= 0,
\end{aligned}
\tag{37}
$$

where R and I denote the real and imaginary part, respectively. The first equation in (37) has the general solution

$$x^{(+)} = \lambda^{(+)}x^{(v)}. \tag{38}$$

To solve the remaining equations of the system (37) we make a partial gauge fixing

$$\lambda^{(+)} = 0 \ \Rightarrow x^{(+)}(d) = 0, \ \ x^{(-)}(d) = 0. \tag{39}$$

The gauge (39) means that the light-like vectors $u_\alpha\bar u_{\dot\alpha}$ and $v_\alpha\bar v_{\dot\alpha}$ of the vector tetrad are tangent vectors to the string's worlsheet. As a result of this gauge choice, the $SO(3,1)$ local symmetry group of the vector frame (35) is reduced to its $SO(1,1)$x$SO(2)$ subgroup and the expansion (36) for $dx_{\alpha\dot\alpha}$ simplifies to

$$dx_{\alpha\dot\alpha} = x^{(u)}(d)u_\alpha\bar u_{\dot\alpha} + x^{(v)}(d)v_\alpha\bar v_{\dot\alpha}. \tag{40}$$

and Eqs.(37) become

$$
\begin{aligned}
x^{(v)} \wedge \varsigma_I &= 0,\\
x^{(u)} \wedge x^{(v)} + \varsigma_R^{(u)} \wedge x^{(v)} + \varsigma_R^{(v)} \wedge x^{(u)} &= 0,\\
x^{(u)} \wedge \varsigma_I^{(v)} - x^{(v)} \wedge \varsigma_I^{(u)} &= 0.
\end{aligned}
\tag{41}
$$

In the gauge (39) the 2-forms $\Sigma_\alpha{}^\beta$ and $\tilde\Sigma^{\dot\beta}{}_{\dot\alpha}$ (21) and $\det(\Sigma_\alpha{}^\beta)$ take the form

$$
\begin{aligned}
\Sigma_\alpha{}^\beta &= -\frac{1}{2}(u_\alpha v^\beta + v_\alpha u^\beta)x^{(u)} \wedge x^{(v)},\\
\tilde\Sigma^{\dot\beta}_{\dot\alpha} &= \frac{1}{2}(\bar u^{\dot\beta}\bar v_{\dot\alpha} + \bar v^{\dot\beta}\bar u_{\dot\alpha})x^{(u)} \wedge x^{(v)} = 0,\\
\det(\Sigma_\alpha{}^\beta) &= \det(\partial_\mu x^m\partial_\nu x_m)(d\tau\wedge d\sigma)^2\\
&= -\frac{1}{4}(x^{(u)} \wedge x^{(v)})^2\\
&= -\frac{1}{4}(\varsigma_R^{(u)} \wedge x^{(v)} + \varsigma_R^{(v)} \wedge x^{(u)})^2
\end{aligned}
\tag{42}
$$

As a result of further analysis of the equations (41) we are able to rewrite the expansions (36) as

$$
\begin{aligned}
dx_{\alpha\dot\alpha} &= x^{(u)}(d)u_\alpha\bar u_{\dot\alpha} + x^{(v)}(d)v_\alpha\bar v_{\dot\alpha},\\
dz_{\alpha\beta} &= (ax^{(u)} + bx^{(v)})u_\alpha u_\beta\\
&+ \left[cx^{(u)} + (1+\bar a)x^{(v)}\right] v_\alpha v_\beta\\
&+ (f_R x^{(u)} + g_R x^{(v)})(u_\alpha v_\beta + u_\beta v_\alpha),
\end{aligned}
\tag{43}
$$

where

$$
\begin{aligned}
\varsigma^{(u)} &= ax^{(u)} + bx^{(v)},\\
a &= a_R + ia_I, \quad b = b_R + ib_I,\\
\varsigma^{(v)} &= cx^{(u)} + dx^{(v)},\\
c &= c_R + ic_I, \quad d = 1 + \bar a,\\
\varsigma_R &= f_R x^{(u)} + g_R x^{(v)},\\
f_I &= 0, \quad \varsigma_I = g_I x^{(v)}.
\end{aligned}
\tag{44}
$$

To solve the equations $\delta S/\delta z_{\alpha\beta} = 0$ (33) we expand du_α and dv_α in the dyad basis

$$
\begin{aligned}
du_\alpha &= \varphi^{(u)}(d)u_\alpha + \varphi^{(v)}(d)v_\alpha,\\
dv_\alpha &= \psi^{(u)}(d)u_\alpha + \psi^{(v)}(d)v_\alpha
\end{aligned}
\tag{45}
$$

and substitute (45) into Eqs.(36)

$$[du_\alpha\wedge(dx\bar u)_\beta - u_\alpha(dx\wedge d\bar u)_\beta] + (\alpha \leftrightarrow \beta) = 0. \tag{46}$$

As a result we find the equations

$$
\begin{aligned}
(\varphi^{(u)} + \bar\varphi^{(u)}) \wedge x^{(v)} = 0,\\
\varphi^{(v)} \wedge x^{(u)} = \varphi^{(v)}) \wedge x^{(v)} = 0,
\end{aligned}
\tag{47}
$$

which have the following general solution

$$
\begin{aligned}
Re\,\varphi^{(u)} &\equiv \varphi_R^{(u)} = \alpha_R^{(u)}x^{(v)},\\
\varphi^{(v)}(d) &= 0
\end{aligned}
\tag{48}
$$

with $\alpha_R^{(u)}(\tau,\sigma)$ an arbitrary real function.

We now turn to the equations $\delta S/\delta x_{\alpha\dot\beta}=0$ in (37). Using the solutions of Eqs. $\delta S/\delta u_\alpha=0$ in (43) and of $\delta S/\delta z_{\alpha\beta}=0$ in (48) it may be written

$$\begin{aligned}(\varphi_I^{(u)}+\alpha_R^{(u)}c_I x^{(u)})\wedge x^{(v)} &= 0,\\ \alpha_R^{(u)}c_R x^{(v)}\wedge x^{(u)} &= 0\end{aligned}\qquad(49)$$

Eqs. (49) have two sets of solutions. The first set is

$$\begin{aligned}\varphi_I^{(u)} &= \alpha_I^{(u)}x^{(v)},\\ \alpha_R^{(u)} &= 0\end{aligned}\qquad(50)$$

and the second set is

$$\begin{aligned}\varphi_I^{(u)} &= -\alpha_R^{(u)}c_I x^{(u)}+\alpha_I^{(u)}x^{(v)},\\ c_R &= 0,\end{aligned}\qquad(51)$$

where $\alpha_I^{(u)}(\tau,\sigma)$ is an arbitrary function.

Having found the general solutions (43), (48) and (50) or (51) to the equations of motion we now also have to analyse the integrability conditions for the expansions (43) and (46). These integrability conditions will play the role of dynamical equations for the string.

5. Solution of the integrability conditions for the $dx_{\alpha\dot\alpha}$, du_α and dv_α expansions

The integrability conditions (IC) $d\wedge dx_{\alpha\dot\alpha}=0$ for the $dx_{\alpha\dot\alpha}-$ expansion (43) are

$$\begin{aligned}d\wedge x^{(u)}+2\alpha_R^{(u)}x^{(v)}\wedge x^{(u)}=0,\\ d\wedge x^{(v)}+2\psi_R^{(v)}\wedge x^{(v)}=0,\\ x^{(v)}\wedge\bar\psi^{(u)}=0,\\ x^{(v)}\wedge\psi^{(u)}=0,\end{aligned}\qquad(52)$$

where we have used (50). It follows directly that

$$\psi^{(u)}=\tilde\Theta^{(u)}x^{(v)},\qquad(53)$$

where $\tilde\Theta^{u}$ is an arbitrary function The IC for the $(du,dv)-$ differential expansions (46) may be written as

$$\begin{aligned}d\wedge\varphi_I^{(u)}=0,\\ d\wedge\varphi_R^{(u)}\equiv d\wedge(\alpha_R^{(u)}x^{(v)})=0,\\ d\wedge\psi^{(v)}=0,\\ d\wedge(\tilde\Theta^{(u)}x^{(v)}+(i\varphi_I^{(u)}-\psi^{(v)})\wedge\tilde\Theta^{(u)}x^{(v)}=0,\end{aligned}\qquad(54)$$

where $\tilde\Theta^{(u)}$ is

$$\tilde\Theta^{(u)}=e^{-(i\tilde\alpha_I^{(u)}-\tilde\Theta^{(v)})}\Theta^{(u)}(\tau,\sigma)\qquad(55)$$

The two sets of equations IC (52) and (54) may be combined into the relations

$$\begin{aligned}\varphi_I^{(u)}=d\tilde\alpha_I^{(u)},\ \varphi^{(v)}=0,\\ \varphi_R^{(u)}=\alpha_R^{(u)}x^{(v)}=d\tilde\alpha_R^{(u)},\\ \psi^{(u)}=e^{-(i\tilde\alpha_I^{(u)}-\tilde\Theta^{(v)})}\Theta^{(u)}x^{(v)},\\ \psi^{(v)}=d\tilde\Theta^{(v)}\end{aligned}\qquad(56)$$

and into the simple system

$$\begin{aligned}&d\wedge(e^{2\tilde\alpha_R^{(u)}}x^{(u)})=0,\\ &d\wedge(\varepsilon^{2\tilde\Theta_R^{(v)}}x^{(v)})=0,\\ &d\wedge(\Theta^{(u)}x^{(v)})=0,\end{aligned}\qquad(57)$$

where $\tilde\alpha_I^{(u)}(\tau,\sigma)$,$\tilde\Theta^{(v)}(\tau,\sigma)$ are arbitrary functions.

In addition the integrability conditions

$$d\wedge dz_{\alpha\beta}=0,\qquad(58)$$

needs to be analysed and the solutions (50) and (51) should be taken into account.

6. Solution of the integrability conditions for the $dz_{\alpha\beta}$ expansion

If we use (44), (45), (48) and (56) the $dz-$ IC's may be written

$$\begin{aligned}d\wedge\left(e^{2\tilde\alpha^{(u)}}ax^{(u)}+e^{2\tilde\alpha^{(u)}}bx^{(v)}\right)+&\\ 2e^{(2\tilde\alpha^{(u)}+i\tilde\alpha_I^{(u)}+\tilde\Theta^{(v)})}\Theta^{(u)}f_R x^{(v)}\wedge x^{(u)} &= 0,\\ d\wedge\left(e^{2\tilde\Theta^{(v)}}cx^{(u)}+e^{2\tilde\Theta^{(v)}}(1+\bar a)x^{(v)}\right) &= 0,\\ d\wedge\left(e^{2(\tilde\alpha^{(u)}+\tilde\Theta^{(v)})}[f_R x^{(u)}+gx^{(v)}]\right)+&\\ e^{(2\tilde\alpha_R^{(u)}+i\tilde\alpha_I^{(u)}+3\tilde\Theta^{(v)})}\Theta^{(u)}cx^{(v)}\wedge x^{(u)} &= 0\end{aligned}\qquad(59)$$

Thus, the final part of our analysis concerns the solution of the equatioqns (59) and (57) and together with the solutions (52) or (53). Here we shall solve Eqs. (59) and (57) choosing the solution (50) which prescribes that

$$\begin{aligned}\alpha_R^{(u)} &= \tilde\alpha_R^{(u)}=0,\\ \varphi_I^{(u)} &= \alpha_I^{(u)}x^{(v)}=d\tilde\alpha_I^{(u)}.\end{aligned}\qquad(60)$$

In that case Eqs.(57) reduces to the equations

$$\begin{aligned} d\wedge x^{(u)} &= 0, \\ d\wedge(e^{2\tilde{\Theta}_R^{(v)}} x^{(v)}) &= 0, \\ d\wedge(\Theta^{(u)} x^{(v)}) &= 0. \end{aligned} \quad (61)$$

Returning to IC (59) we note that the last equation may be satisfied if the arbitrary functions f_R, g, $\tilde{\Theta}_I^{(v)}$, $c\Theta^{(u)}$ are fixed by the relations

$$\begin{gathered} e^{2\tilde{\Theta}_R^{(v)}} f_R = \mathcal{A}_f = const, \\ \tilde{\Theta}_I^{(v)} + \tilde{\alpha}_I^{(u)} = \triangle_0 = const, \\ g_R = \mathcal{A}_R = const, \ g_I = \mathcal{A}_I = const, \\ c\Theta^{(u)} = 0. \end{gathered} \quad (62)$$

The last equation in (62) has two solutions

$$\Theta^{(u)} = 0 \Longrightarrow \psi^{(u)} = 0 \quad (63)$$

and

$$c = 0. \quad (64)$$

The case (63) seems to be the simplest for further analysis. In that case the IC (59) are reduced to the form

$$\begin{gathered} \Theta^{(u)} = \psi^{(u)} = 0, \\ d\wedge\left(e^{2i\tilde{\alpha}_I^{(u)}}[ax^{(u)} + bx^{(v)}]\right) = 0, \\ d\wedge\left(e^{2(\tilde{\Theta}_R^{(v)} - i\tilde{\alpha}_I^{(u)})}[cx^{(u)} + (1+\bar{a})x^{(v)}]\right) = 0. \end{gathered} \quad (65)$$

Still there is a possibiulity to furter simplify the reduced IC (65) fixing an arbitrariness in the definitions of the functions a, b and c by the relations

$$\begin{gathered} b = b_0\, e^{2(\tilde{\Theta}_R^{(v)} - i\tilde{\alpha}_I^{(u)})}, \quad b_0 = const., \\ a = a_0 e^{-2i\tilde{\alpha}_I^{(u)}}, \quad a_0 = \bar{a}_0 = const, \\ c = c_0\, e^{-2(\tilde{\Theta}_R^{(v)} - i\tilde{\alpha}_I^{(u)})}, \quad c_0 = const. \end{gathered} \quad (66)$$

Then, as a result of (61), we find the single IC

$$d\tilde{\alpha}_I^{(u)} \wedge x^{(v)} = 0, \quad (67)$$

which is identically satisfied using the second relation in equation (60).

Thus, the total set of the IC's under consideration is reduced to the set

$$\begin{aligned} d\wedge x^{(u)} &= 0, \\ d\wedge(e^{2\tilde{\Theta}_R^{(v)}} x^{(v)}) &= 0, \\ d\tilde{\alpha}_I^{(u)} &= \alpha_I^{(u)} x^{(v)}, \end{aligned} \quad (68)$$

which has the following solutions

$$\begin{gathered} x^{(u)} = d\eta^{(u)}, \\ x^{(v)} = e^{-2\tilde{\Theta}_R^{(v)}} d\eta^{(v)}, \\ d\tilde{\alpha}_I^{(u)} = \alpha_I^{(u)} e^{-2\tilde{\Theta}_R^{(v)}} d\eta^{(v)}, \end{gathered} \quad (69)$$

where $\eta^{(u)}(\tau,\sigma)$ and $\eta^{(v)}(\tau,\sigma)$ are atbitrary functions. The last IC in (69) is easily satisfied choosing

$$\alpha_I^{(u)} = e^{2\tilde{\Theta}_R^{(v)}} \beta'(\eta^{(v)}) \quad (70)$$

which gives the following representation for $\tilde{\alpha}_I^{(u)}$

$$\tilde{\alpha}_I^{(u)} = \beta(\eta^{(v)}(\tau,\sigma)), \quad (71)$$

where $\beta(\eta^{(v)})$ is an arbitrary functions.

In the next section we shall discuss an example of string motion arising from the above equations.

7. An example of the string motion in the extended space-time

To get an example of string motion let us fix the function $\beta(\eta^{(v)})$ in the solution (71) by the condition $\beta'(\eta^{(v)}) = 1$ which gives

$$\tilde{\alpha}_I^{(u)} = \eta^{(v)}, \ \ \alpha_I^{(u)} = \alpha_0 e^{2\tilde{\Theta}_R^{(v)}} \quad (72)$$

and then the total set of the integrability conditions is satisfied.

The final equations (43) corresponding to the solutions (72) and (69) and describing the string dynamics take the simple form

$$\begin{aligned} dx_{\alpha\dot{\alpha}} &= u_\alpha \bar{u}_{\dot{\alpha}} d\eta^{(u)} + v_\alpha \bar{v}_{\dot{\alpha}} e^{-2\tilde{\Theta}_R^{(v)}} d\eta^{(v)}, \\ dz_{\alpha\beta} &= e^{-2i\eta^{(v)}}(a_0 d\eta^{(u)} + b_0 d\eta^{(v)}) u_\alpha u_\beta \\ &\quad + e^{-2\tilde{\Theta}_R^{(v)}}[c_0 e^{2i\eta^{(v)}} d\eta^{(u)} \\ &\quad + (1 + a_0 e^{2i\eta^{(v)}}) d\eta^{(v)}] v_\alpha v_\beta \\ &\quad + e^{-2\tilde{\Theta}_R^{(v)}}(\mathcal{A}_f d\eta^{(u)} + \mathcal{A} d\eta^{(v)})(u_\alpha v_\beta + u_\beta v_\alpha), \\ du_\alpha &= i d\eta^{(v)} u_\alpha, \\ dv_\alpha &= (-i d\eta^{(v)} + d\tilde{\Theta}_R^{(v)}) v_\alpha. \end{aligned} \quad (73)$$

To analyse Eqs.(73) we note that the spinor subset has the general solution

$$\begin{gathered} u_\alpha = u_{0\alpha} e^{i\eta^{(v)}(\tau,\sigma)}, \\ v_\alpha = v_{0\alpha} e^{-i\eta^{(v)}(\tau,\sigma) + \tilde{\Theta}_R^{(v)}(\tau,\sigma)}, \end{gathered} \quad (74)$$

where $u_{0\alpha}$ and $v_{0\alpha}$ are arbitrary constant spinors. Substituting the solutions (74) into the constraints (6) results in the same constraints for both dyads u_0^α and v_0^α

$$u_0^\alpha v_{0\alpha} = 1, \quad u_0^\alpha u_{0\alpha} = v_0^\alpha v_{0\alpha} = 0, \tag{75}$$

and implies

$$\tilde{\Theta}_R^{(v)} = 0. \tag{76}$$

So, we find that for the presented example the whole dynamics of the dyads u_α and v_α is reduced to phase transformations

$$\begin{aligned} u_\alpha &= u_{0\alpha} e^{i\eta^{(v)}(\tau,\sigma)}, \\ v_\alpha &= v_{0\alpha} e^{-i\eta^{(v)}(\tau,\sigma)}. \end{aligned} \tag{77}$$

It follows from the solutions (77) that

$$\begin{aligned} u_\alpha \bar{u}_{\dot\alpha} &= u_{0\alpha}\bar{u}_{0\dot\alpha} = const, \\ v_\alpha \bar{v}_{\dot\alpha} &= v_{0\alpha}\bar{v}_{0\dot\alpha} = const. \end{aligned} \tag{78}$$

The conditions (78) essentially simplify equation (73) for the string world vector $x_{\alpha\dot\alpha}$ transforming it into the equation

$$dx_{\alpha\dot\alpha} = d\eta^{(u)} u_{0\alpha}\bar{u}_{0\dot\alpha} + d\eta^{(v)} v_{0\alpha}\bar{v}_{0\dot\alpha} \tag{79}$$

which has the following general solution

$$x_{\alpha\dot\alpha} = x_{0\alpha\dot\alpha} + \eta^{(u)} u_{0\alpha}\bar{u}_{0\dot\alpha} + \eta^{(v)} v_{0\alpha}\bar{v}_{0\dot\alpha}, \tag{80}$$

where $\eta^{(u)}$ and $\eta^{(v)}$ are arbitrary functions on the worldsheet. According to Eqs.(69), which now take the form

$$x^{(u)} = d\eta^{(u)}, \;\; x^{(v)} = d\eta^{(v)}, \tag{81}$$

one finds from (42) and (81) that

$$\begin{aligned} &\det(\Sigma_\alpha{}^\beta) = \det(\partial_\mu x^m \partial_\nu x_m)(d\tau \wedge d\sigma)^2 \\ &= -\frac{1}{4}(x^{(u)} \wedge x^{(v)})^2 = -\frac{1}{4}(d\eta^{(u)} \wedge d\eta^{(v)})^2, \end{aligned} \tag{82}$$

so that the expected condition (31) $\det\Sigma \neq 0$, is satisfied for the solution (80) and the induced worldsheet metric of the tensionless string becomes regular. As the vectors

$$\begin{aligned} \frac{\partial x_{\alpha\dot\alpha}}{\partial\eta^{(u)}} &= u_{0\alpha}\bar{u}_{0\dot\alpha} \equiv (\sigma_m)_{\alpha\dot\alpha} A^m, \\ \frac{\partial x_{\alpha\dot\alpha}}{\partial\eta^{(v)}} &= v_{0\alpha}\bar{v}_{0\dot\alpha} \equiv (\sigma_m)_{\alpha\dot\alpha} B^m \end{aligned} \tag{83}$$

tangent to the worldsheet are constant light-like vectors we conclude that the worldsheet (10) lies in the plane spanned by the light-like vectors A^m and B^m

$$\begin{aligned} &x^m = x_0^m + A^m \eta^{(u)}(\tau+\sigma) + B^m \eta^{(v)}(\tau-\sigma), \\ &\qquad A^m A_m = B^m B_m = 0, A^m B_m = 1. \end{aligned} \tag{84}$$

and we see that the central charge coordinates do not excitate the transverse oscillations of the x^m-coordinates. But what about the z^{mn}-coordinates?

By analogy with the previous case taking into account the solutions (74) simplifies Eqs.(73) for the central charge coordinates $z_{\alpha\beta}$ to the form

$$\begin{aligned} &dz_{\alpha\beta} = (a_0 d\eta^{(u)} + b_0 d\eta^{(v)}) u_{0\alpha}u_{0\beta} \\ &+ \left(c_0 d\eta^{(u)} + (e^{-2i\eta^{(v)}} + a_0) d\eta^{(v)}\right) v_{0\alpha}v_{0\beta} \\ &+(\mathcal{A}_f d\eta^{(u)} + \mathcal{A} d\eta^{(v)})(u_{0\alpha}v_{0\beta} + u_{0\beta}v_{0\alpha}) \end{aligned} \tag{85}$$

and the general solution of (85) is

$$\begin{aligned} &z_{\alpha\beta} = z_{0\alpha\beta} + (a_0\eta^{(u)} + b_0\eta^{(v)}) u_{0\alpha}u_{0\beta} \\ &+(c_0\eta^{(u)} + a_0\eta^{(v)} - 2ie^{-2i\eta^{(v)}}) v_{0\alpha}v_{0\beta} \\ &+(\mathcal{A}_f\eta^{(u)} + \mathcal{A}\eta^{(v)})(u_{0\alpha}v_{0\beta} + u_{0\beta}v_{0\alpha}). \end{aligned} \tag{86}$$

The solutions (80) and (86) can be interpreted as solutions of the two-dimensional wave equations

$$\begin{aligned} &\ddot{\mathrm{x}}_{\alpha\dot\alpha} - x''_{\alpha\dot\alpha} = 0, \\ &\ddot{\mathrm{z}}_{\alpha\beta} - z''_{\alpha\beta} = 0, \end{aligned} \tag{87}$$

which may be presented in the form

$$\ddot{\mathrm{X}}_{\mathcal{M}} - X''_{\mathcal{M}} = 0, \tag{88}$$

where $X^{\mathcal{M}} = (x^m, z^{mn})$ are generalized coordinates in the extended D=10 space-time. The wave equation (88) implies that the coordinates z^{mn} have an interpretation similar to that for the x^m namely as additional string coordinates in the generalized D=10 space. After the substitution of the solution (86) into the representation

$$z_{mn} = \frac{i}{4}[z_\alpha{}^\beta(\sigma_{mn})_\beta{}^\alpha + \bar{z}_{\dot\alpha}{}^{\dot\beta}(\tilde\sigma_{mn})^{\dot\alpha}{}_{\dot\beta}] \tag{89}$$

for z^{mn} we find the appearance of the spinorial structures $(u_0\sigma_{mn}u_0)$, $(v_0\sigma_{mn}v_0)$ and $(u_0\sigma_{mn}v_0)$,

which correspond to the spin degreees of freedom distributed along the string worldsheet. These spin factors are multiplied by the functions which are solutions of the wave equation (88). Therefore, we may understand the solution (86) as describing a spin wave process associated with the z^{mn} degrees of freedom.

8. Conclusion

We have suggested a new model for strings embedded into $D = 4$ space-time extended by 6 additional coordinates z^{mn} corresponding to the tensor central charge Z^{mn}. In studying the simplest case of a tensionless string we found that presence of the z^{mn} coordinates lifts the degeneration of the worldsheet metric typical for the tensionless string. We found a solution of the model which gives an example of string dynamics in the extended $D = (4+6)$-dimensional space. For this solution the x^m-coordinates do not develop transverse oscillations. To understand if this result is general or if the x^m coordinates also can oscillate we need to analyse the general solution to the integrability conditions derived here or/and to extend the minimal model. On the other hand, the evolution of the z^{mn} coordinates may be understood as a spin wave process assosiated with an excitation of spin degrees of freedom distributed along the string.

The next step is to study the general case of the tensile string described by the general action (14) with the spin-tensor p^{ab} given by (9).

The supersymmetrized version of the model is found by going from the differentials dx^m and dz^{mn} to the supersymmetric invariant differential Cartan forms Π^m and Π^{mn} in the general representation (14) or (16) and adding a Chern-Simons three-form [26]. Therefore, an example of a superstring action in $D = 4$ space-time extended by the coordinates z^{mn} is

$$\tilde{S} = i\kappa \int \{[(\Pi_m \wedge \Pi_n - 8\Pi_{ml} \wedge \Pi_n{}^l)\gamma^{mn} - 4ic\Pi^l \wedge \Pi_{lm}\gamma^m]_a{}^b p^a{}_b + \Lambda + (Chern - Simons)\}, \qquad (90)$$

where the constant c may be zero. The extended superalgebra with tensorial central charges Z^{mn} arising from the superstring action (89) (and/or its brane-generalizations) may contain new information on the preserved supersymmetry. An investigation of this is in progress.

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